Module Name: Probability distribution of a two-option system (even and odd) with the help of rolling dice

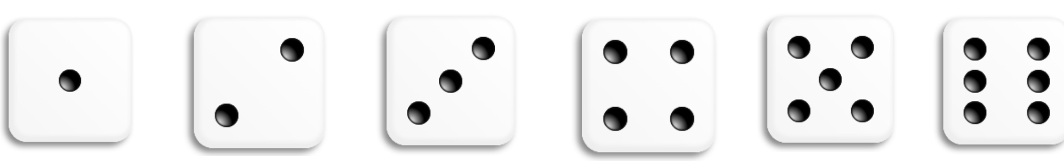
1. Learning Outcomes

Upon completion of this module the learner will be able to: -

- Remember the definition of probability.
- Understand the phenomenon frequency distribution.
- Apply to any number of dices ranging from 2 to 12.
- Analyze to identify the probability distribution of a two-option system
- Evaluate the mean and standard deviation of experimental frequencies.
- Create animation/visual effects for statistical systems.

2. Introduction

To calculate the probability distribution of a two-option system (even and odd) by tossing dice, we must consider the probabilities of all possible outcomes. For example, there are six possible outcomes if we roll one die: 1, 2, 3, 4, 5, and 6.



Prof Sheetal Malviya, Assistant Professor and Dr. Netram Kaurav, Assistant Professor	Department of Physics, Govt. Holkar (Model Autonomous) Science College, Indore (MP)-452001 India
Prof V. P. Verma, Assistant Professor	Department of Physics, Govt. Degree College, Sanwer, Indore (MP)- 453551 India

Each possible outcome has a probability of 1/6. These outcomes can be divided into two categories: even and odd. Three outcomes are even: 2, 4, and 6. There are three odd results: 1, 3, and 5. The probability of an even result is either 3/6 or 1/2. The probability of getting an odd outcome is also 3/6 or 1/2. Consequently, the probability distribution of a two-option system (even and odd) determined by tossing (rolling) a single die is as follows:

Outcome	Probability
Even	1/2
Odd	1/2

The probability of occurrence of an event is equal to the ratio of the number of cases in which the event occurs to the total number of cases. i.e.,

$$Probability\ of\ an\ event = \frac{number\ of\ cases\ in\ which\ the\ event\ occurs}{Total\ number\ of\ cases}$$

Let *x* be the number of cases in which the event occurs and *y* be the number of cases in which the event does not occur, then total number of cases will be ($x + y$) and the probability of occurrence of event is

$$p = \frac{x}{x+y}$$

While probability of non-occurrence of event is

$$q = \frac{y}{x+y}$$

Obviously, $p + q = 1$ or $q = 1 - p$, i.e., the total probability is always equal to 1.

3. Procedure/Methodology

Consider the following scenario in order to comprehend the probability distribution of a two-option system involving the tossing of dice. Suppose we have a six-sided die with the numbers 1 through 6. We wish to ascertain the probability distribution of the result being even or odd. In this instance, the die contains three even numbers (2, 4, and 6) and three odd numbers (1, 3, and 5). Since the die is impartial, each of these outcomes has an equal chance of occurring.

To compute the probability distribution, we must ascertain the possibility of each possible outcome. Given that there are six equally probable outcomes (the numbers 1 through 6), the probability of any particular number landing face-up is 1/6. Here, we will consider to have the probability of even number on the top of the face.

Let's consider member of particles to be 'n' and we want to distribute them in two boxes A and B. The microstates of these distributions

$(n, o),\ (n-1, 1), (n-2, 2), \ldots\ldots\ldots\ldots\ldots\ldots (n-r,\ r), \ldots\ldots\ldots. (o, n)$

The number of ways that a particular distribution is given by:

$$ {}^{n}c_{r} = \frac{n!}{(n-r)!\,r!} $$

i.e., for (n, o) the ways = ${}^{n}c_{o}$

For $(n, 1)$ … … … … ….. ….= ${}^{n}c_{1}$ … … and so on.

Therefore, the total number of ways of all possible distribution is

$$^{n}c_{r} + {}^{n}c_{1} + {}^{n}c_{2} + \cdots \ldots \ldots {}^{n}c_{r} + \cdots \ldots \ldots + {}^{n}c_{n} = 2^{n}$$

Above distribution fulfil in the case of two option system (even and odd), and ‘n’ is the number of dice.

Here, some important steps if we consider 8 dices.

$$^{n}c_{r} = \frac{n!}{(n-r)!\,r!}$$

No of dice having even on the face, *r*:

$$^{8}c_{0} = \frac{8!}{(8-0)!\,0!} = 1$$

$$^{8}c_{1} = \frac{8!}{(8-1)!\,1!} = 8$$

$$^{8}c_{2} = \frac{8!}{(8-2)!\,2!} = 28$$

$$^{8}c_{3} = \frac{8!}{(8-3)!\,3!} = 56$$

$$^{8}c_{4} = \frac{8!}{(8-4)!\,4!} = 70$$

$$^{8}c_5 = \frac{8!}{(8-5)!\,5!} = 56$$

$$^{8}c_6 = \frac{8!}{(8-6)!\,6!} = 28$$

$$^{8}c_7 = \frac{8!}{(8-7)!\,7!} = 8$$

$$^{8}c_8 = \frac{8!}{(8-1)!\,8!} = 1$$

Therefore, the total number of rolling/tosses = 1+8+28+56+70+56+28+8+1

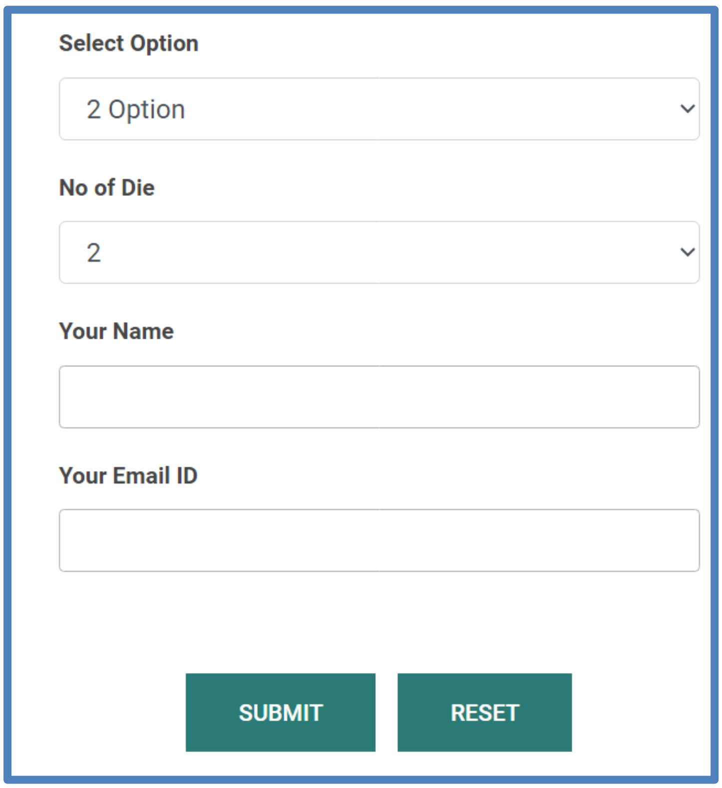
= 256

= 2^8

= 2X2X2X2X2X2X2X2

Step I: Select the option, i.e., two face option and then choose no of dices. Here we have provided 2 to 12 dices.

Output of interface



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Prof Sheetal Malviya, Assistant Professor and Dr. Netram Kaurav, Assistant Professor	Department of Physics, Govt. Holkar (Model Autonomous) Science College, Indore (MP)-452001 India
Prof V. P. Verma, Assistant Professor	Department of Physics, Govt. Degree College, Sanwer, Indore (MP)- 453551 India

Step II: Now generating the frequencies with help of random function, depending upon the number of dices (Total number of toss or rolling out will be (No of option) $^{\text{no of dices}}$. Here it will be 2^n, n being the number of dices.

Output of interface

Option = 2 No of Die = 4

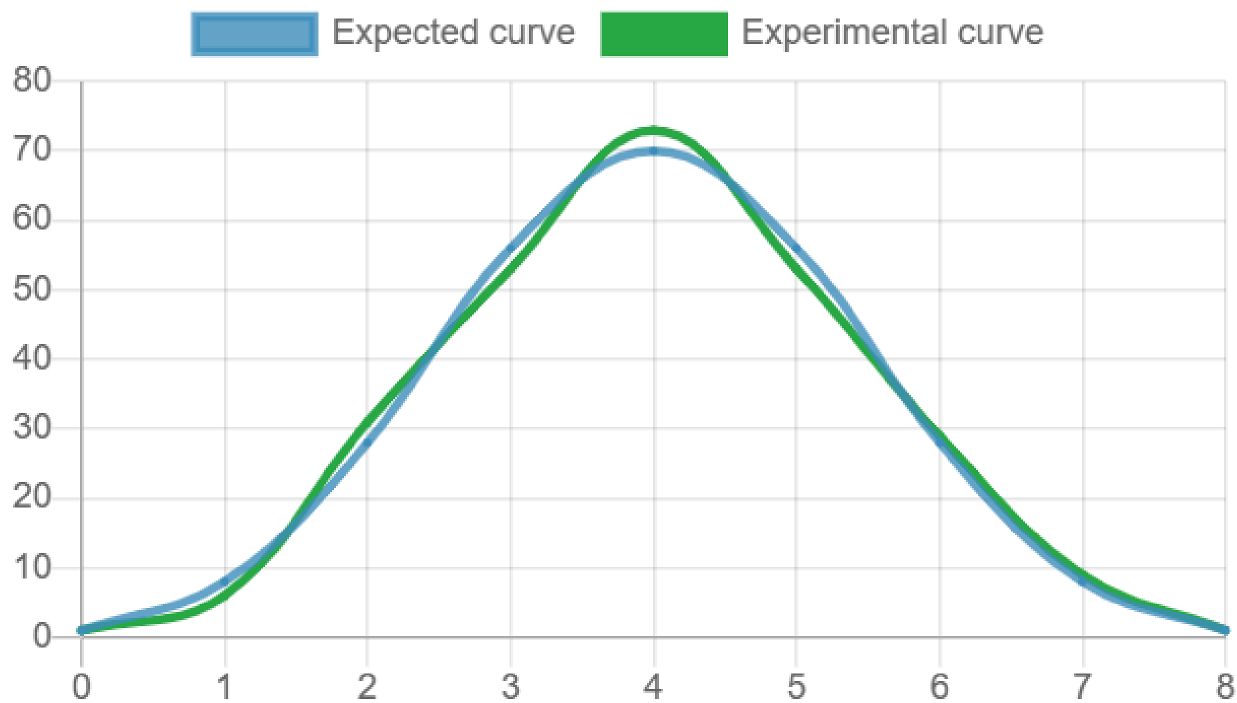
1425	1446	3364	2542
3335	1445	1443	2615
1252	5524	1124	3123
2432	2325	3121	4324

In the above case, we have chosen 4 dices, therefore, the matrix is be 2^4= 16. If there are 8 dices then it will be 2^8= 256.

Step III: Later, following table will be prepared based on the number of dice.

No of dice having even No (r) on the face	Expected frequency $\Omega = {}^{n}C_{r}$	Experimental frequency (f)	f.r	$(r-\bar{r})$	$(r-\bar{r})^2$	f. $(r-\bar{r})^2$
0						
1						
2						
. **.** **.**						
	Σf		Σf.r			Σ f. $(r-\bar{r})^2$

Step IV: Now a plot will be prepared with expected and experimental frequencies.



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Step V: Calculations:

1). Mean of expected frequencies = np = 8x(1/2)= 4

2). Standard deviation of expected frequencies = $\sqrt{npq}$ = $\sqrt{8\times\frac{1}{2}\times\frac{1}{2}}$ = 1.141

3). Mean of experimental frequencies $\bar{r}$ = $\frac{\sum fr}{\sum f}$

4). Standard deviation of experimental frequencies = $\sqrt{\frac{\sum f\left(r-\bar{r}\right)^2}{\sum f}}$

Results:

- It is supposed that the mean and standard deviation of expected frequencies and of experimental frequencies are nearly the same i.e., both agree quite well.
- The experimental probability distribution curve agrees with the expected distribution curve.
- More the number of events more is agreement in experimental and expected distribution frequencies.